LINEAR CIRCUIT ANALYSIS (EED) – U.E.T. TAXILA | 12

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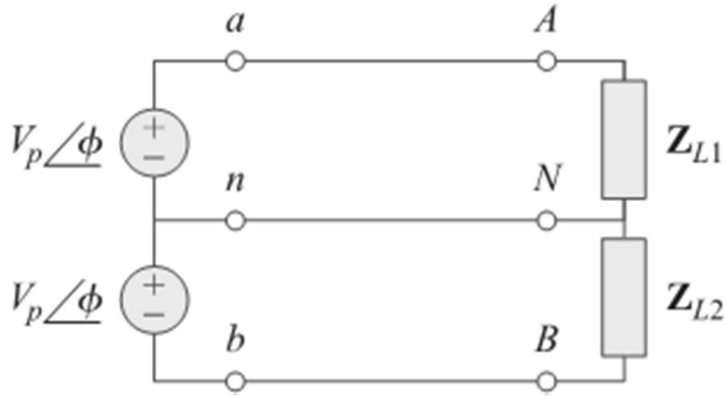
INTRODUCTION

A single-phase ac power system consists of a generator connected through a pair of wires to a load.

A single-phase two wire system consists of one phase wire and one neutral wire.

A single-phase three wire system consists of two identical sources (equal in magnitude and same phases).

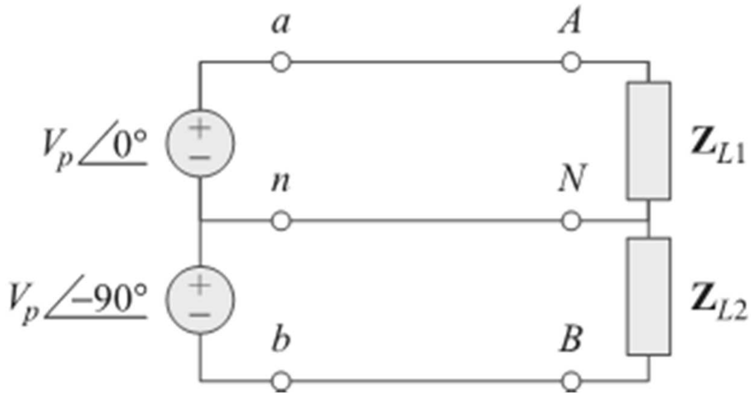
INTRODUCTION



A system in which the ac sources operate at the same frequency but different phases is known as Poly-Phase System.

A two-phase three wire system contains two identical sources but different in phase.

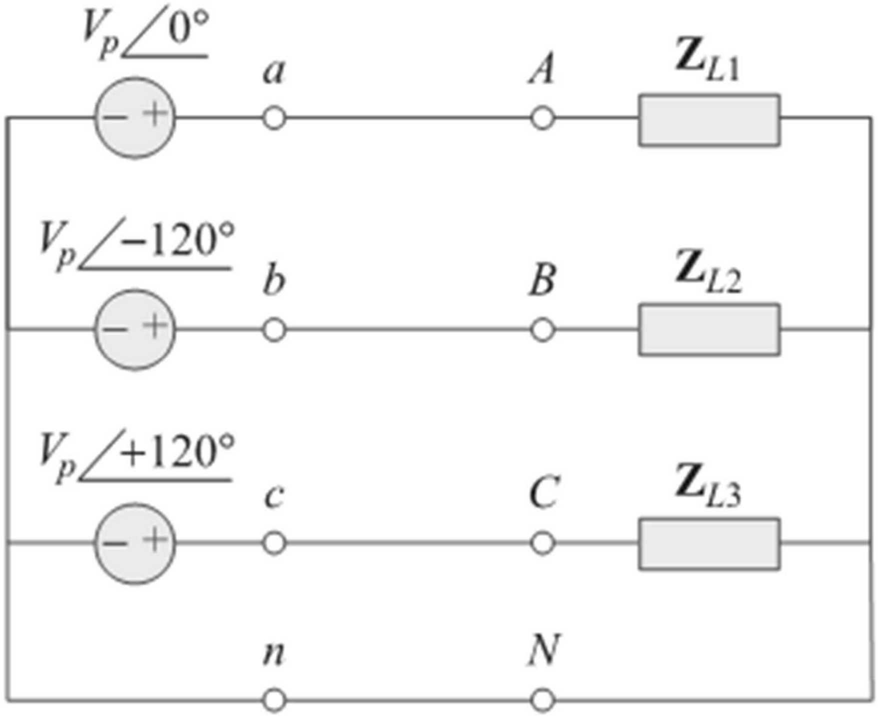
INTRODUCTION



A three-phase four wire system contains three identical sources but different in phase.

In poly-phase systems, voltages of certain sources lead or lag the others.

INTRODUCTION



INTRODUCTION

Three-phase systems are important for three reasons;

First, all electric power is generated and distributed in three-phase.

Second, the instantaneous power in three-phase systems is constant.

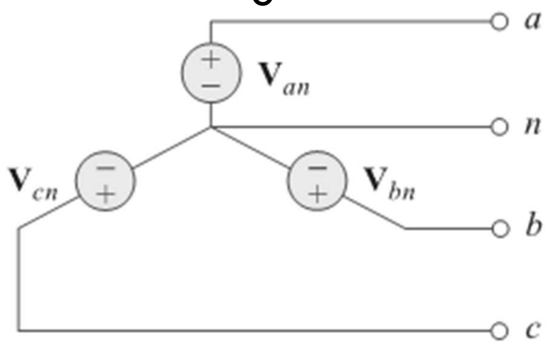
Third, for the same amount of power, the three-phase system is more economical than single phase for the reason that amount of wire required for three-phase system is less than single-phase.

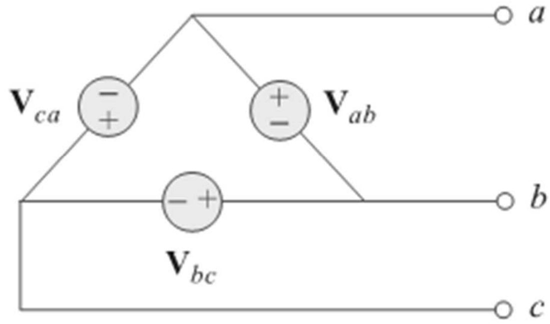
BALANCED THREE-PHASE VOLTAGES

A typical three-phase system consists of three voltage sources connected to loads by three or four wires (transmission lines).

A three-phase system is equivalent to three single-phase circuits.

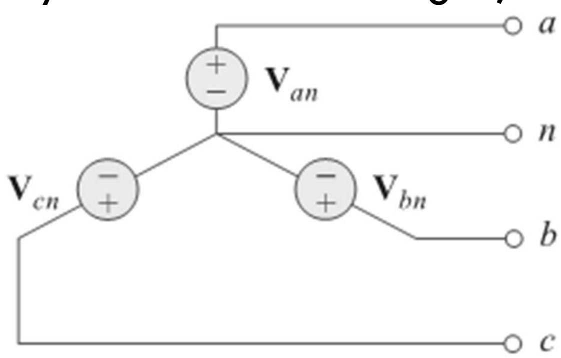
Three voltage sources can be connected in wye or delta.





BALANCED THREE-PHASE VOLTAGES

First, consider wye-connected voltages;



The voltages V_{an}, V_{bn}, V_{cn} are respectively between lines a, b, c and the neutral line n.

These voltage are called phase voltages.

BALANCED THREE-PHASE VOLTAGES

Balanced Phase Voltages are equal in magnitude and are out of phase with each other by 120°.

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

Mathematically; (where V_p is rms value)

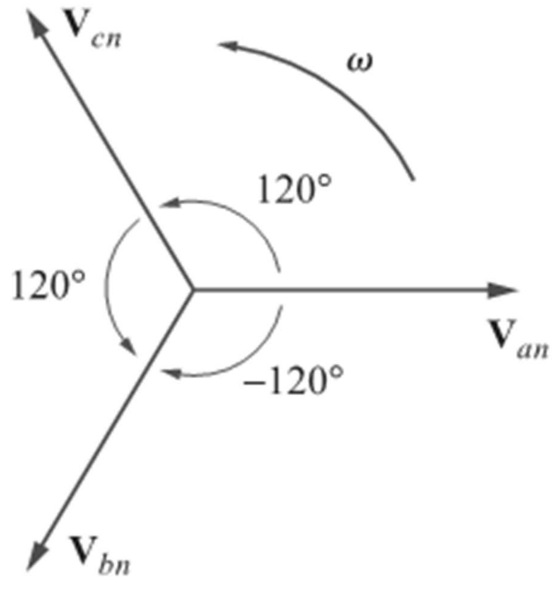
$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -120^\circ$$

$$\mathbf{V}_{cn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

BALANCED THREE-PHASE VOLTAGES

It is evident that V_{an} leads V_{bn} which in turn leads V_{cn}, this is abc and is called positive phase sequence.



BALANCED THREE-PHASE VOLTAGES

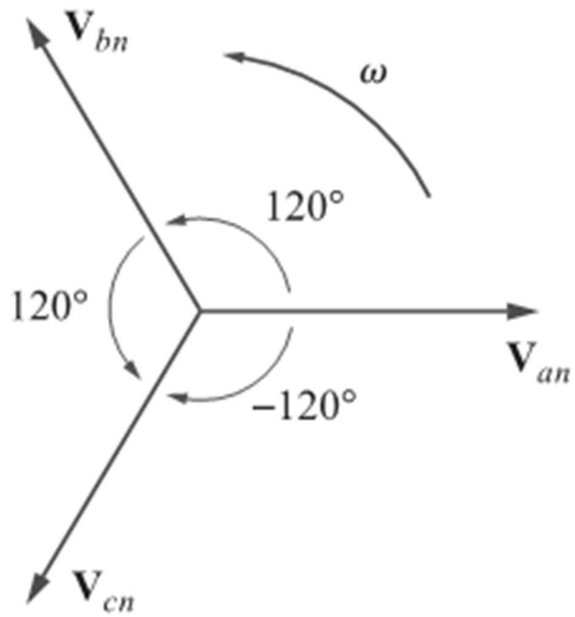
Consider the set of three-phase voltages;

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -240^\circ = V_p\angle +120^\circ$$

For this phase sequence, V_{an} leads V_{cn} which in turn leads V_{bn}.

This phase sequence is acb and is called negative phase sequence.

BALANCED THREE-PHASE VOLTAGES

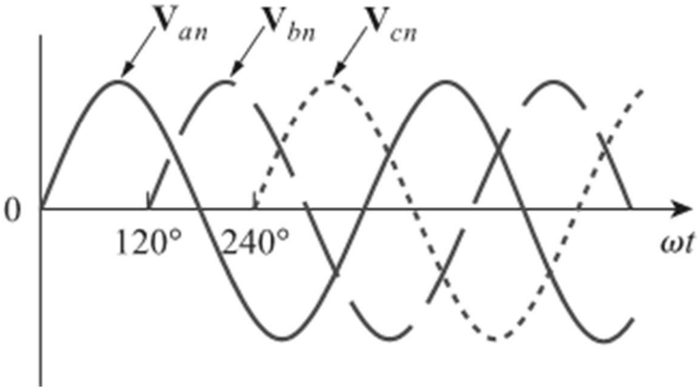


BALANCED THREE-PHASE VOLTAGES

The Phase Sequence is the time order in which the voltages pass through their respective maximum values.

The phasor sum of three-phase voltages is always zero.

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = V_p\angle 0^\circ + V_p\angle -120^\circ + V_p\angle +120^\circ$$
$$= V_p(1.0 - 0.5 - j0.866 - 0.5 + j0.866)$$
$$= 0$$



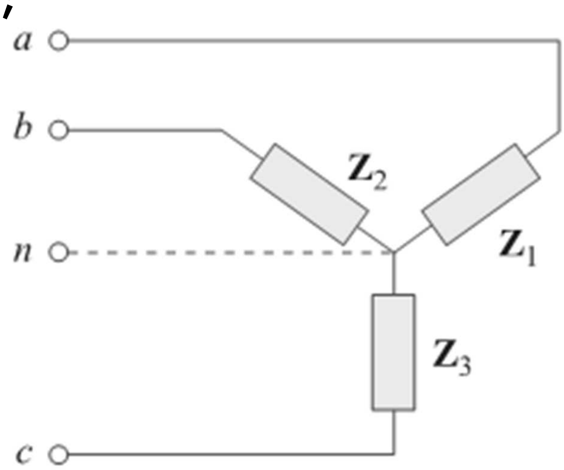
BALANCED THREE-PHASE VOLTAGES

The loads can also be connected as three-phase load.

A Balanced Load is one in which the phase impedances are equal in magnitude and in phase.

Balanced wye-connected load;

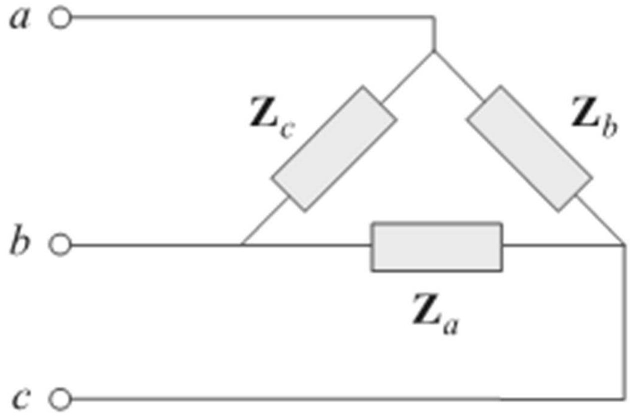
$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$



BALANCED THREE-PHASE VOLTAGES

Balanced delta-connected load;

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$



Relation b/w wye and delta impedances;

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

BALANCED THREE-PHASE VOLTAGES

Since both the three-phase source and three-phase load can be either wye or delta connected, there are four possible connections;

Y-Y connection

Y-Δ connection

Δ-Δ connection

Δ-Y connection

PROBLEMS

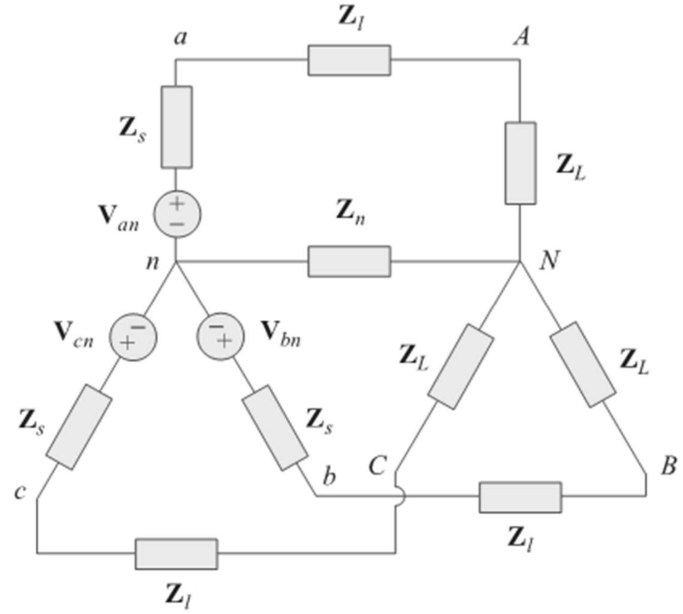
Determine the phase sequence of the set of voltages?

$$v_{an} = 200\cos(\omega t + 10^\circ)$$
$$v_{bn} = 200\cos(\omega t - 230^\circ)$$
$$v_{cn} = 200\cos(\omega t - 110^\circ)$$

(acb)

BALANCED WYE-WYE CONNECTION

A Balanced Y-Y System is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.



BALANCED WYE-WYE CONNECTION

The total load impedance per phase may be regarded as the sum of source impedance, line impedance and load impedance.

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L$$

The source and line impedances are very small as compared to load impedance, assuming;

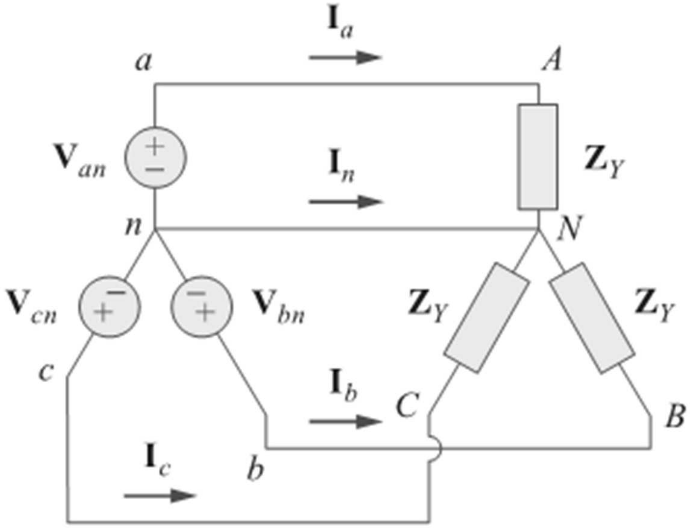
$$\mathbf{Z}_Y = \mathbf{Z}_L$$

Consider positive sequence phase voltages;

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

BALANCED WYE-WYE CONNECTION



The line-to-line voltages or simply line voltages V_{ab}, V_{bc}, V_{ca} can be related as;

$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0^\circ - V_p\angle -120^\circ$$

BALANCED WYE-WYE CONNECTION

$$\mathbf{V}_{ab} = V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_p\angle 30^\circ$$

Similarly;

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90^\circ$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210^\circ$$

Thus, the magnitude of the line voltages V_L is $\sqrt{3}$ times the magnitude of the phase voltages V_p.

$$V_L = \sqrt{3}V_p$$

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

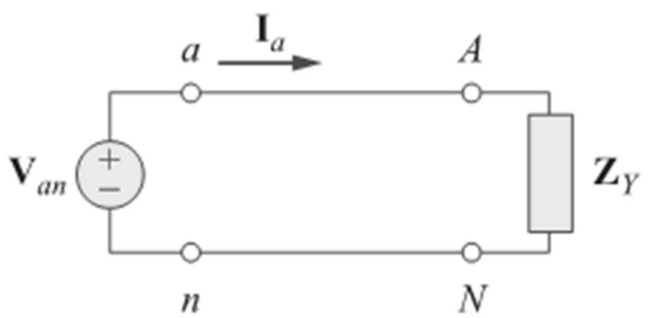
$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$

BALANCED WYE-WYE CONNECTION

The line current is the current in each line while the phase current is the current in each phase of source or load.

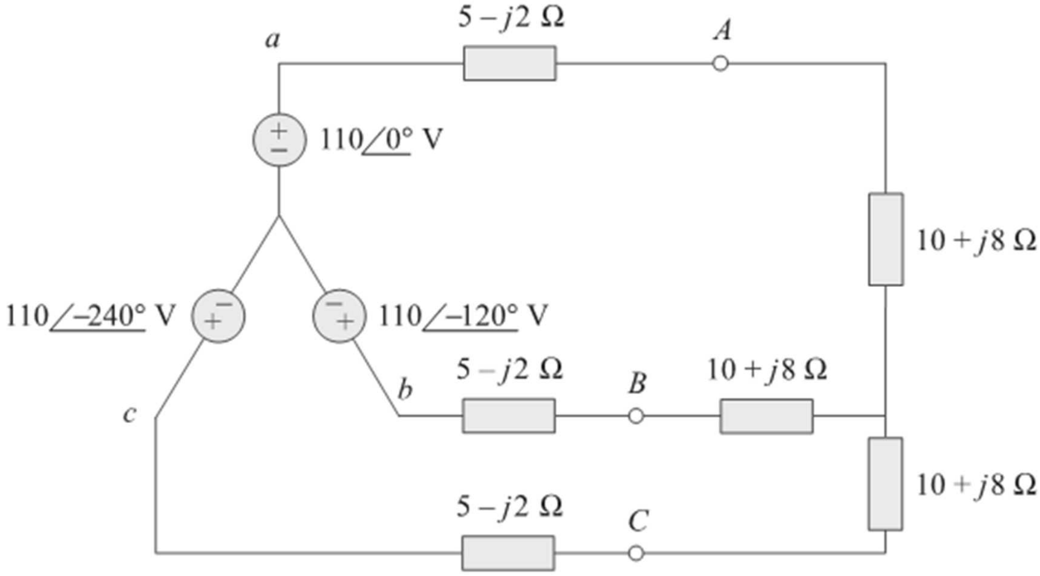
It is evident that line current is the same as the phase current in Y-Y system.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$



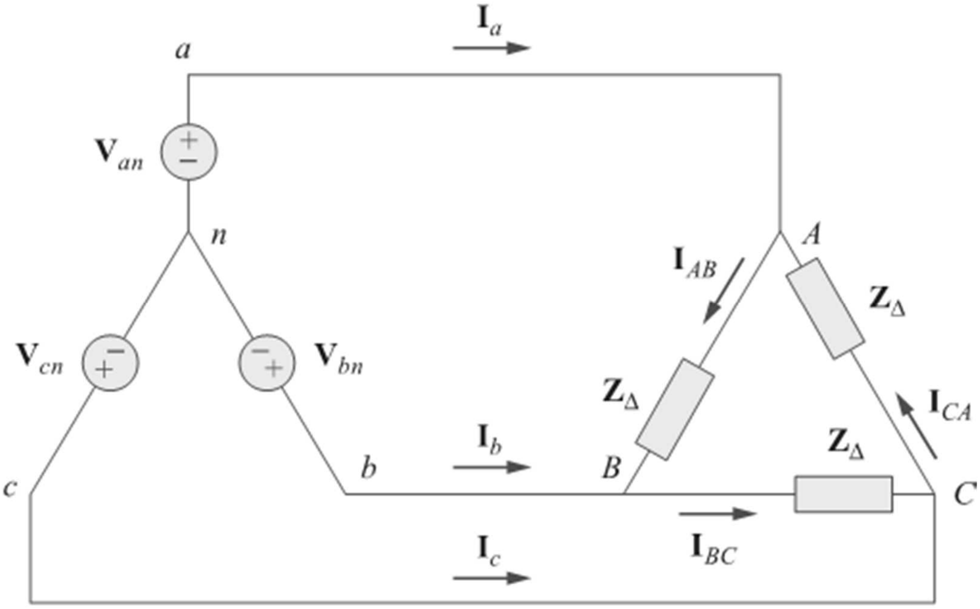
PROBLEMS

Calculate line currents?



BALANCED WYE-DELTA CONNECTION

A Balanced Y-Δ System consists of a balanced Y-connected source feeding balanced Δ-connected load.



BALANCED WYE-DELTA CONNECTION

Consider positive sequence phase voltages;

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

For Y-connected source;

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ = \mathbf{V}_{BC}$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle -150^\circ = \mathbf{V}_{CA}$$

For Δ-connected load;

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta}$$

BALANCED WYE-DELTA CONNECTION

Applying KCL at nodes A, B and C;

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle -240^\circ)$$
$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$$

Thus, the magnitude of the line currents I_L is $\sqrt{3}$ times the magnitude of the phase currents I_p.

$$I_L = \sqrt{3}I_p$$

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

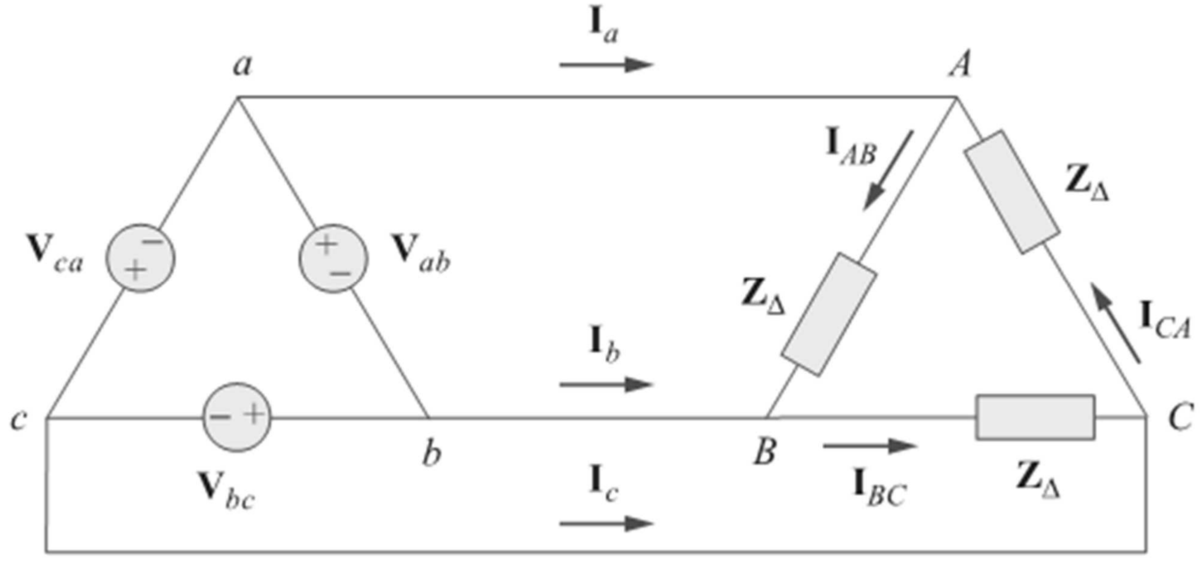
$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

PROBLEMS

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100\angle 10^\circ$ V is connected to a Δ-connected balanced load $(8 + j4)\ \Omega$ per phase. Calculate the phase and line currents.

BALANCED DELTA-DELTA CONNECTION

A Balanced Δ-Δ System is one in which both the balanced source and balanced load are Δ-connected.



BALANCED DELTA-DELTA CONNECTION

Consider positive sequence phase voltages;

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$

$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

For Δ-connected source;

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

For Δ-connected load;

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta} = \frac{\mathbf{V}_{ab}}{Z_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{Z_\Delta} = \frac{\mathbf{V}_{bc}}{Z_\Delta}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{Z_\Delta} = \frac{\mathbf{V}_{ca}}{Z_\Delta}$$

BALANCED DELTA-DELTA CONNECTION

Applying KCL at nodes A, B and C;

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

$$\begin{aligned}\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} &= \mathbf{I}_{AB}(1 - 1\angle -240^\circ) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ\end{aligned}$$

Thus, the magnitude of the line currents I_L is $\sqrt{3}$ times the magnitude of the phase currents I_p.

$$I_L = \sqrt{3} I_p$$

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

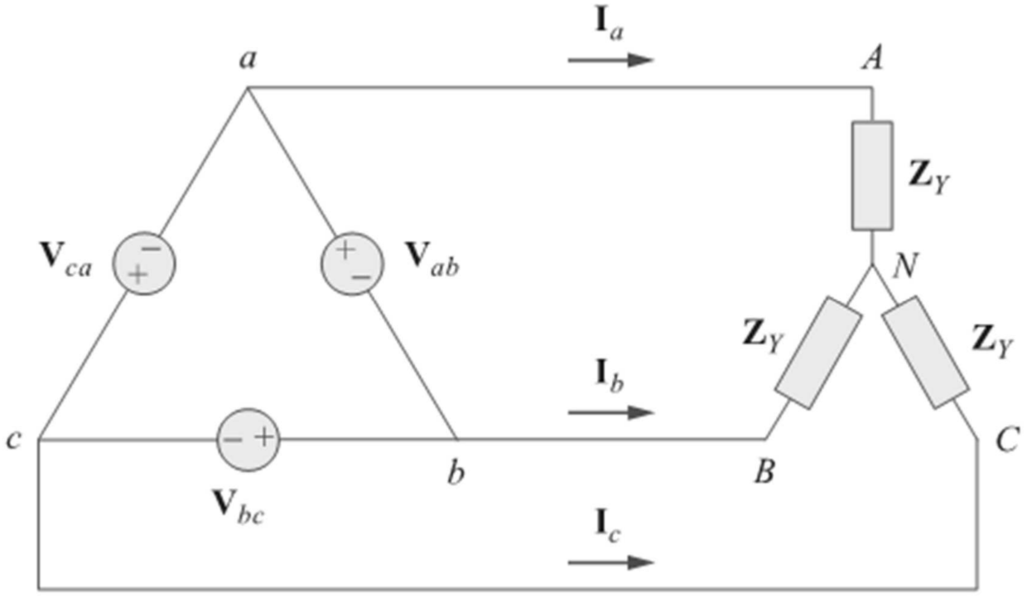
$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

PROBLEMS

A balanced Δ-connected load having an impedance $20 - j15\ \Omega$ is connected to a Δ-connected, positive-sequence generator having $\mathbf{V}_{ab} = 330\underline{/0^\circ}$ V. Calculate the phase currents of the load and the line currents.

BALANCED DELTA-WYE CONNECTION

A Balanced Δ-Y System consists of a balanced Δ-connected source feeding a balanced Y-connected load.



SUMMARY

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

SUMMARY

Connection	Phase voltages/currents	Line voltages/currents
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

POWER IN A BALANCED SYSTEM

In time domain, phase voltages for Y-connected load;

$$v_{AN} = \sqrt{2}V_p \cos \omega t, \qquad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120°)$$
$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120°)$$

Load Impedance; $\mathbf{Z}_Y = Z\underline{/\theta}$

Phase currents;

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta), \qquad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120°)$$
$$i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120°)$$

Instantaneous power;

$$p = p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c$$

POWER IN A BALANCED SYSTEM

Substituting values of voltages and currents;

$$p = 3V_pI_p \cos\theta$$

Active, reactive and apparent power per phase;

$$P_p = V_pI_p \cos\theta$$
$$Q_p = V_pI_p \sin\theta$$
$$S_p = V_pI_p$$

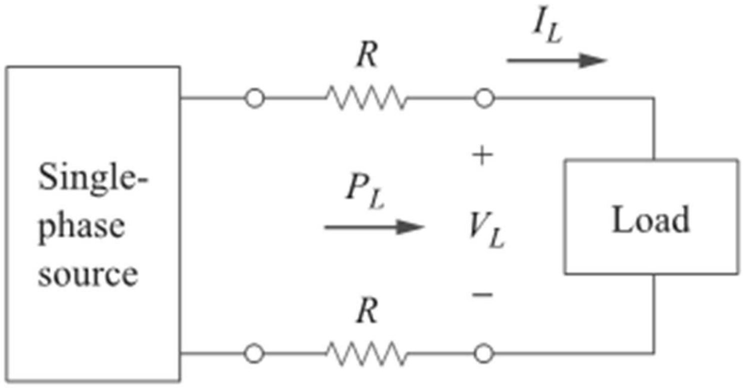
Three-phase active, reactive and apparent power;

$$P = \sqrt{3}V_LI_L \cos\theta \qquad Q = \sqrt{3}V_LI_L \sin\theta$$

POWER IN A BALANCED SYSTEM

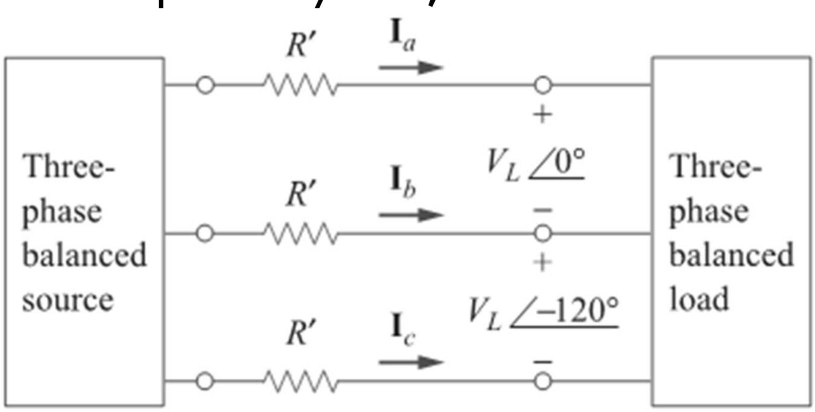
A major advantage of three-phase system for power distribution is that the three-phase system uses a lesser amount of wire than single-phase system for same power.

Consider single-phase system; $P_{\text{loss}} = 2I_L^2R = 2R\frac{P_L^2}{V_L^2}$



POWER IN A BALANCED SYSTEM

Consider three-phase system;



$$P'_{\text{loss}} = 3(I'_L)^2R' = 3R'\frac{P_L^2}{3V_L^2} = R'\frac{P_L^2}{V_L^2}$$

POWER IN A BALANCED SYSTEM

Dividing;

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2R}{R'}$$

$$R = \rho\ell/\pi r^2 \text{ and } R' = \rho\ell/\pi r'^2,$$

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2r'^2}{r^2}$$

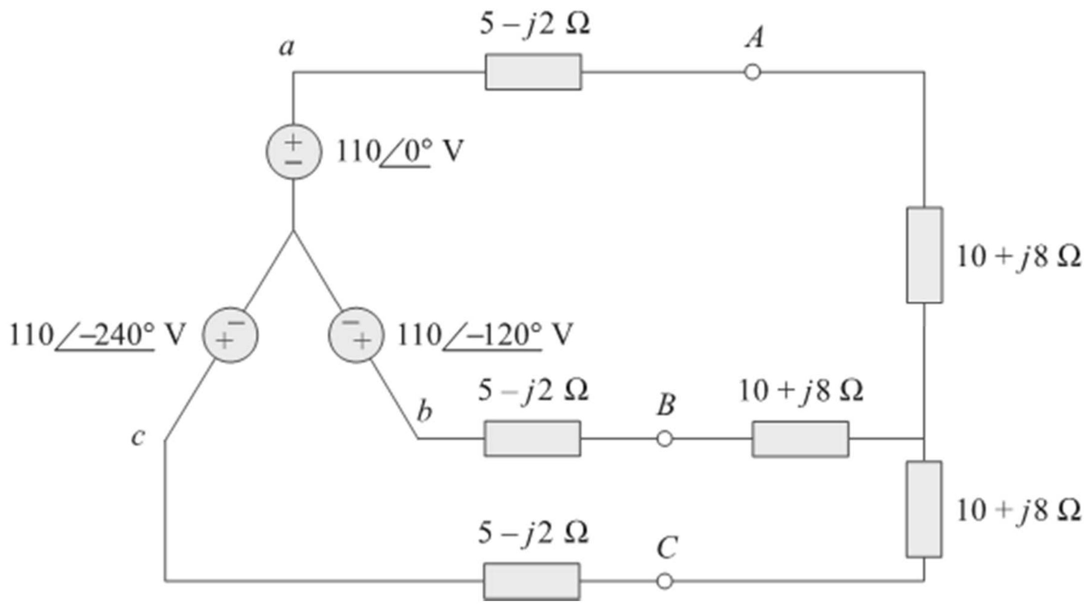
Ratio of material;

$$\frac{\text{Material for single-phase}}{\text{Material for three-phase}} = \frac{2(\pi r^2\ell)}{3(\pi r'^2\ell)} = \frac{2r^2}{3r'^2}$$

$$= \frac{2}{3}(2) = 1.333$$

PROBLEMS

Find total active, reactive and complex power at source and load?



REFERENCES

Fundamentals of Electric Circuits (4th Edition)

Charles K. Alexander, Matthew N. O. Sadiku

Chapter 12 – Three-Phase Circuits (12.1 – 12.7)

Exercise Problems: 12.1 – 12.50

Do exercise problem yourself.